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A Graph-Based Loss Allocation Framework for Transactive Energy Markets in Unbalanced Radial Distribution Networks

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***Abstract*— Future distributed transactive energy markets are envisioned to integrate multiple entities located mainly at the distribution level of the grid, so that consumers and prosumers can trade power either directly with the upstream wholesale energy market or through local pricing mechanisms (e.g. through "*peer-to-peer*" contracts or by forming "*energy communities*"). In such market environments, a transparent loss allocation framework is required to guarantee economic efficiency and fairness. Nevertheless, the unbalanced power flow nature associated with the distribution networks should be fundamentally integrated in the loss allocation process. To this end, this paper presents a graph-based loss allocation framework that harmonizes the physical attributes of the distribution grid with the underlying financial transactions in distributed market settings. The latter is achieved via representing distribution networks as multi-layered radial graphs. The proposed loss allocation framework is tested on an actual LV feeder that is experiencing rapid growth in DG applications.**

***Index Terms*—Distributed energy resources, Loss allocation, Peer-to-peer trading, Unbalanced distribution networks, Transactive energy.**

NOMENCLATURE

e	Line
n	Node
k	Phase layer
N	Neutral layer
NN	Total number of nodes
E	Total number of lines
a_e^k	The ratio of the line's resistance on phase k over the square of the network's nominal phase voltage [Ω/V²]
$I_p^{n,k}$	Nodal real power injection/absorption on phase k [kW]
$I_q^{n,k}$	Nodal reactive power injection/absorption on phase k [kVAr]
$P_e^{n,k}$	Contribution of nodal real power $I_p^{n,k}$ to the real power flow of line e on phase k [kW]
$Q_e^{n,k}$	Contribution of nodal reactive power $I_q^{n,k}$ to the reactive power flow of line e on phase k [kVAr]
J_e^k	Complex power flow through line e on phase k [kW+jkVAr]
JP_e^k	Real power flow through line e on phase k [kW]
JQ_e^k	Reactive power flow through line e on phase k [kVAr]
L_e^k	Losses incurred on line e on phase k [W]
Lp_e^k	Losses due to real power flow on line e on phase k [W]
Lq_e^k	Losses due to reactive power flow on line e on phase k [W]
TR	Transaction
$AL_{TR}^{X_kY_k}$	Allocated losses to TR^{th} transaction for using the line connecting nodes X and Y on phase k [W]
TAL_{TR}^k	Sum of allocated losses to TR^{th} transaction for using the lines on phase k [W]
P_{TR}	The amount of real power of the TR^{th} transaction [kW]
Q_{TR}	The amount of reactive power of the TR^{th} transaction [kVAr]

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I. INTRODUCTION

TRANSACTIVE energy (TE) is an emerging concept describing the organized management of the interaction between multiple active market entities (e.g., smart consumer devices, distributed generating sources, storage devices, etc.) based on the underlying value of the services exchanged [1]. In other words, within TE environments, dynamic pricing signals will act as the effective means to the coordination of service exchanges [1].

In this regard, TE concepts offer a vision of retail energy markets that may operate either as direct extensions of wholesale markets down to the distribution level, or as distributed markets acting in parallel to central wholesale markets [2]. Specifically, distributed market environments are envisaged as local pricing mechanisms that take into account local demand, generation and network characteristics and thus enable the various consumers and prosumers to exchange services in a bilateral, *peer-to-peer* manner or by forming *Energy Communities* (ECs); that is, *groups* of local consumers and prosumers that produce, consume and share resources in a joint fashion [3], [4]. Hence, different actors may trade energy via locally varying prices determined by diverse objectives and capabilities [1]–[8].

However, it should be borne in mind that power exchanges between buyers and sellers inevitably incur losses. This entails the extra energy amounts and costs –over and above the local net demand– that must be produced and subsequently recovered by each market entity. To this extent, a transparent loss allocation framework (to be applied by independent authorities,

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e.g. the market or the distribution system operators) should ideally offer economic efficiency and impartiality towards all participants.

A review of loss allocation methods for the transmission and distribution level can be found in [9] and [10] respectively. In addition, Table I provides a summary of the available literature pertaining to the various loss allocation methods. As per the references provided in Table I, there exists a wide variety of loss allocation methods, each with its own characteristics and rationale.

With regard to transaction-tracing loss allocation methods in particular, it is noted that these should entail: a) explicit modeling of transaction paths and b) direct quantification of each transaction's impact on the incurred losses. In this context, the emergence of TE markets and peer-to-peer trading raises a series of issues (addressed below in detail) pertaining to allocating losses and associated charges that have not yet been addressed in the relevant literature.

TABLE I. LITERATURE SUMMARY

Allocation method	Description	Transmission level	Distribution level
Pro rata	Loss allocation based on the active power levels of generators and/or consumers	[9]	[10]–[12]
Marginal	Loss allocation based on the impact of an incremental change of power on the total incurred losses	[13]–[15]	[16], [17]
Flow-tracing	Loss allocation based on the topological solution of the contribution of each generator/load to the flows and losses of each line	[18], [19]	[20]
Circuit-based	Loss allocation based on the network characteristics and related circuit theories	[21], [22]	[23]–[25]
Transaction-tracing	Loss allocation based on the impact of each transaction on the losses of each branch of the network	[26]–[31]	Present work

A. *Specific contributions of this work*

With regard to the above discussion and Table I, this paper attempts to address the following questions pertaining to distributed TE markets:

1. Which is the relationship between the upstream and local markets in terms of losses and their allocation?
2. How does the unbalanced nature of distribution networks affect the peer-to-peer transactions' modelling and the subsequent loss allocation?
3. How can the physical topology of distribution networks be harmonized with the underlying financial transactions and subsequently integrated in the loss allocation process?

The first question relates to which entity is responsible for providing the incurred losses to the local network. Bearing in mind that losses are determined by the *real-time* interactions between buyers and sellers, it should be noted that the default supplier of reserve energy is in effect the provider of the incurred losses as well [13]. For distribution networks in particular, this role has been traditionally assumed by the upstream market; specifically, the extra energy required to supply the incurred losses is fed through the point of connection between the upstream and local market (e.g. MV/LV substation). To this extent, from a loss allocation standpoint, the framework proposed in this paper provides an explicit answer to which portion of the total incurred losses –provided by the *upstream* market– is caused by each transaction that takes place within the *local* market.

The second question that is addressed in this paper is the unbalanced nature of loads and Distributed Generation (DG) units, especially at the low voltage (LV) level. The unbalanced conditions suggest that the underlying transactions may, for example, entail single-phase generators serving three-phase consumers; therefore, particular emphasis should be given in appropriately modelling the various types of transactions (either local *peer-to-peer* trades or direct exchanges with the upstream market). This is crucially important in order to accurately capture the actual grid interaction of buyers and sellers and subsequently integrate it in the allocation process.

Finally, the fundamental characteristics of the topology of distribution networks are also taken into account. With respect to the latter, distribution networks are conventionally designed as weakly meshed and are operated as radial or tree networks [32]. Therefore, this paper proposes a unified framework that harmonizes power flows and financial transactions by taking advantage of the fact that within radial or tree feeders the path from one node to another is unique [33].

In order to assure that these objectives are achieved, the remainder of this paper is organized as follows: firstly, a concise description of the derivation of power flows in unbalanced distribution networks is provided. This pertains to the calculation of line flows and associated incurred losses not only for the phase conductors but also for the neutral conductors. Subsequently, a graph-theoretical approach regarding the topology of three-phase, four-wire (3Ph-4W) distribution networks is presented and discussed. The latter is performed in order to facilitate the transaction modelling by deriving the transaction paths from first principles. Particular emphasis is given in the multi-layered topology of distribution networks. Next, the proposed loss allocation framework is presented based on the adopted modelling approach. Finally, the paper concludes with a numerical case study pertaining to *peer-to-peer* trading and *Energy Communities* based on an actual LV feeder. The case study highlights the importance of accurately modelling the unbalanced conditions at the distribution level with regard to losses and their allocation.

II. POWER FLOW MODELLING

Distribution networks have been traditionally designed to serve (normally unbalanced) single-, two- or three-phase loads. Therefore, their most common setup entails three phase conductors and a fourth neutral wire that is used to provide a pathway back to the source [34]. Moreover, such networks are presently required to accommodate the increasing penetration of DG units, which may also be connected as single-phase generation sources. This can potentially increase *net* load unbalances and, consequently, neutral flows [35].

For simplicity but without loss of generality, the power flows through a three phase-four wire (3Ph-4W) network can be approximated via the algorithm provided in Table II [32], [34], [36]. To this end, within Table II, two variables are utilized at each node (n) of each phase (k: A, B or C); namely, the real and reactive power, $I_p^{n,k}$ and $I_q^{n,k}$ respectively. The sign of these variables corresponds to their direction and therefore depends on whether the customer at each node imports power from or injects power to the grid. More specifically, the following convention is used in this paper: power (either real or reactive) is signed positive if it is imported at a node. Conversely, power is signed negative if it is injected at a node.

Table II uses a subtlety regarding line flows and losses. Particularly, the orthogonal decomposition of flows is used to separate the (complex) power flow (J_e) through a line (e) into real and reactive components (JP_e and JQ_e respectively) [22]; thus, real and reactive line flows can be determined and treated separately. This is utilized within Table II whereby the quantification of the real and reactive power flow of each line per each phase is determined via the superposition of the respective nodal real and reactive power absorptions/injections ($I_p^{n,k}$ and $I_q^{n,k}$ respectively) [22]. In particular, the superposition of nodal real power absorptions/injections determines the lines' real power flows (JP_e^k). Similarly, the superposition of nodal reactive power absorptions/injections determines the lines' reactive power flows (JQ_e^k). However, one should note that the power absorption/injection at node n on phase k contributes to a line's flow on the same phase only if that line belongs to the path connecting the reference node (i.e. the point of connection between the upstream and local market) to node n [22], [32], [34]. This is modelled via variables $P_e^{n,k}$ and $Q_e^{n,k}$ noted in Table II.

TABLE II. ALGORITHM FOR DETERMINING LINE FLOWS AND LOSSES IN 3PH-4W DISTRIBUTION NETWORKS

For *each line e of a total E lines*
  For *each node n of a total NN nodes*
    For *each phase k (i.e. A, B or C),*
    starting from the reference node to reach node n, check if line e is crossed

$$P_e^{n,k} = \begin{cases} I_p^{n,k}, & \text{if yes} \\ 0, & \text{if no} \end{cases}$$

$$Q_e^{n,k} = \begin{cases} I_q^{n,k}, & \text{if yes} \\ 0, & \text{if no} \end{cases}$$

    End
  End

$$JP_e^k = \sum_{n=1}^{NN} P_e^{n,k}$$

$$JQ_e^k = \sum_{n=1}^{NN} Q_e^{n,k}$$

$$J_e^k = JP_e^k + jJQ_e^k$$

$$L_e^k = a_e^k \times (JP_e^k)^2 + a_e^k \times (JQ_e^k)^2 = Lp_e^k + Lq_e^k$$

$$J_e^N = (J_e^A)^* + (J_e^B)^* \times e^{-j120^\circ} + (J_e^C)^* \times e^{-j240^\circ}$$

$$JP_e^N = \mathrm{Re}\{J_e^N\}$$

$$JQ_e^N = \mathrm{Im}\{J_e^N\}$$

$$L_e^N = a_e^N \times (JP_e^N)^2 + a_e^N \times (JQ_e^N)^2 = Lp_e^N + Lq_e^N$$

End

Moreover, for each line (e) per each phase k, losses (L_e^k) are assumed to be equal to $a_e^k (J_e^k)^2$ where a_e^k is determined by the voltage and resistance of the respective line whilst $(J_e^k)^2$ is the square of the (complex) power flowing through that line [23], [36]. Specifically, the parameter $a_e^k = R_e^k / V^2$ where R_e^k is the resistance of line e on phase k whereas V^2 is the square of the network's nominal phase voltage.

With regard to the neutral conductors' flows, the flows of the phase conductors are used in order to derive the flows through each segment of the neutral conductor (J_e^N). Finally, the losses on the neutral conductor are also determined as $a_e^N (J_e^N)^2$. Furthermore, line losses can also be fundamentally separated into those caused by real power flows (Lp_e^k) and those caused by reactive power flows (Lq_e^k) as per (1) [22], [24].

$$\begin{aligned} L_e &= a_e \left|J_e\right|^2 = a_e \left(\sqrt{\mathrm{Re}\{J_e\}^2 + \mathrm{Im}\{J_e\}^2}\right)^2 = \\ &= a_e JP_e^2 + a_e JQ_e^2 = Lp_e + Lq_e \\ &\text{where } a_e JP_e^2 = Lp_e \text{ and } a_e JQ_e^2 = Lq_e \end{aligned} \tag{1}$$

This property is particularly useful from a loss allocation perspective. To make the latter argument more explicit, we take advantage of the fact that JP_e^k and JQ_e^k result from the superposition of the nodal power absorptions/injections (as shown in Table II). Thus, the losses caused by the contribution of each nodal power absorption/injection to each line flow could be quantified as per (2) [22], [24], [25]. Within (2), $AL_e^{n,k}$ is the amount of losses that are allocated to the power absorption/injection of node n on phase layer k for using line e on the same phase layer [22], [24], [25].

$$AL_e^{n,k} = \frac{P_e^{n,k}}{JP_e^k} \times Lp_e^k + \frac{Q_e^{n,k}}{JQ_e^k} \times Lq_e^k \tag{2}$$

III. TRANSACTION MODELLING

Within TE market environments, the import/export activities of customers at each node are associated with one or more transactions. Therefore, the main challenge of the loss allocation process lies in accurately modelling the relationship between these import/export activities and the underlying transactions.

In general, a transaction (TR) is characterized by: a) the real and reactive transacted power amounts (P_{TR} and Q_{TR} respectively), b) the sending node, c) the receiving node and d) the *transaction path*. With regard to the latter and in order to facilitate the loss allocation process, the transaction paths are derived by adopting a graph-theoretical approach for radial or tree networks.

A *path* connecting a specified pair of nodes (x,y) is defined as the sequence of nodes and lines that are traversed in order to reach node y starting from node x [33]. Pertaining to radial or tree graphs, an inherent property is that each path from any node to another is *unique* [33]. For demonstration purposes, Fig. 1 illustrates a small-scale tree graph. With reference to Fig. 1, starting from node B to reach node F, it can be easily deduced that the path is (BA)-(AD)-(DF). Working in reverse, starting from node F to reach node B, the path is (FD)-(DA)-(AB). In other words, the same lines are crossed but at the exact opposite direction. This property of radial and tree graphs is particularly useful in modelling transactions for distribution networks.

However, due to the presence of the neutral conductor, this condition (i.e. path uniqueness) is not directly fulfilled by 3Ph-4W distribution networks, as will be further elaborated in section III.A. To this end, a more comprehensive approach is utilized in order to derive the transaction paths in 3Ph-4W distribution networks.

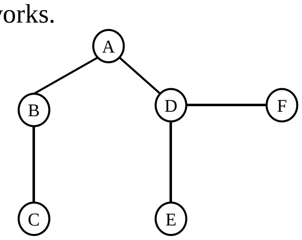

Fig. 1: Small-scale example of a tree graph.

A. Deriving transaction paths in three-phase, four-wire distribution networks

Distribution networks can be notionally perceived as multi-layered graphs [33] consisting of three phase layers and a fourth neutral layer. The latter is shared by all three phases as the layer that completes the circuit. To this extent, 3Ph-4W networks can be transformed into radial or tree graphs simply via the *virtual* separation of the phase layers from the neutral layer. This concept is illustrated in Fig. 2, which is essentially a generalized representation of the tree graph shown in Fig. 1.

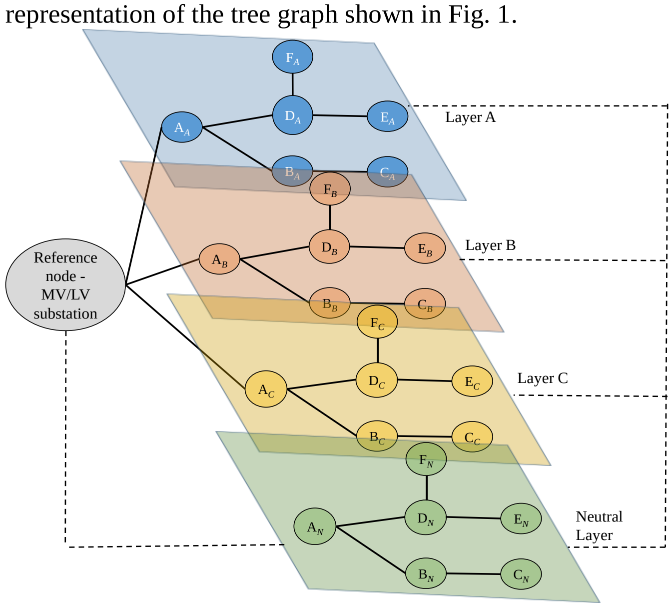

Fig. 2: Illustration of the (virtual) separation of three-phase, four-wire distribution networks into multiple layers.

This (virtual) separation into multiple layers entails that the path from any node to another becomes unique and can be extracted from the graph's topology. To demonstrate the path derivation (in the context of transactive markets), let us assume three distinct examples pertaining to: 1) transactions with the upstream electricity market, 2) peer-to-peer transactions between local nodes on the same layer, and, 3) peer-to-peer transactions between local nodes on different layers. The transaction details are shown in Table III.

TABLE III. TRANSACTION EXAMPLES

Example	P_{TR} (pu)	Q_{TR} (pu)	*From Node*	*To Node*
1	2	0	F_A	A_A
2	1	0	F_A	E_A
3	1	0	F_A	E_B

1. In the first example, it is assumed that node F on layer A is selling 2 units of real power to the upstream market and, therefore, the transaction path is (F_AD_A)-(D_AA_A).
2. In the second example, it assumed that node F on layer A sells 1 unit of real power to node E on the same layer. Now, the transaction path is (F_AD_A)-(D_AE_A).
3. Finally, in the third example, it is assumed that node F on layer A sells 1 unit of real power to node E on layer B. Thus, the transaction path in this case involves two phase layers and is derived as (F_AD_A)-(D_AA_A)-(A_BD_B)-(D_BE_B).

In the three examples mentioned above, the transaction path essentially determines which parts of the grid are used when the transaction takes place. This is the keystone of the subsequent loss allocation process (see section IV).

However, in order for the loss allocation process to be whole, the impact of each transaction on the neutral layer must also be taken into account. Due to the fact that the neutral layer is essentially the pathway that "*completes the circuit*", the path of each transaction is in effect *mirrored* on the neutral layer. That is, if a transaction causes a flow on phase layer k from node x_k to node y_k, then it also causes a mirrored flow from node y_N to node x_N on the neutral layer. To make this process more explicit, the three transaction examples of Table III are revisited below with reference to Fig. 2.

1. In the first example, the transaction path is (F_AD_A)-(D_AA_A) and, therefore, its mirrored path on the neutral layer is (A_ND_N)-(D_NF_N).
2. In the second example, the transaction path is (F_AD_A)-(D_AE_A). Thus, its mirrored path on the neutral layer is (E_ND_N)-(D_NF_N).
3. Finally, in the third example, the transaction path is (F_AD_A)-(D_AA_A)-(A_BD_B)-(D_BE_B). Now, the mirrored path onto the neutral layer is (E_ND_N)-(D_NA_N)-(A_ND_N)-(D_NF_N).

The latter example prompts an important conclusion regarding the neutral layer. Specifically, when a transaction involves more than a single phase layer, this may entail overlaps on the neutral layer. In other words, the mirrored path may include segments of the neutral layer that are used by both phase layers involved in the transaction. This fact should in principle be incorporated in the loss allocation process (see Section IV).

B. Justification of transaction path derivation

In order to justify the path derivation and the mirrored projection of phase flows onto the neutral layer, the flow sensitivity of the network in Fig. 2 is investigated. Initially, it is assumed that the network is operating under balanced conditions. That is, all phase layers exhibit the same real and reactive power characteristics as per Table IV.

TABLE IV. REAL AND REACTIVE POWER AT EACH NODE OF FIG. 2

Node	**Phase Layer A** ($I_p^A + jI_q^A$) **– pu**	**Phase Layer B** ($I_p^B + jI_q^B$) **– pu**	**Phase Layer C** ($I_p^C + jI_q^C$) **– pu**
A	Reference node		
B	1+1j	1+1j	1+1j
C	2+1j	2+1j	2+1j
D	1+0j	1+0j	1+0j
E	3+1j	3+1j	3+1j
F	-4+1j	-4+1j	-4+1j

Utilizing the algorithm provided in Table II, the respective line flows can be calculated per each layer, including the neutral

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layer. These results are provided in Table V. Within Table V, it is shown that neutral flows are initially zero due to the balanced real and reactive net demand characteristics of the phase layers.

TABLE V. REAL AND REACTIVE POWER FLOWS THROUGH EACH LINE OF FIG. 2

Line	Phase Layer A – pu	Phase Layer B – pu	Phase Layer C – pu	Neutral Layer – pu
AB	3+2j	3+2j	3+2j	0
BC	2+1j	2+1j	2+1j	0
AD	0+2j	0+2j	0+2j	0
DE	3+1j	3+1j	3+1j	0
DF	-4+1j	-4+1j	-4+1j	0

Subsequently, we marginally increase the exchanged power amount of Example 3 (see Table III) by an infinitesimal amount (e.g. 0.1 pu) and discern the *changes* (ΔJ_e^k) in all line flows of the network. The corresponding flows after the marginal increase are shown in Table VI whilst the respective changes in flows (i.e. the difference between flows before and after the marginal increase) are provided in the parentheses of the same table.

TABLE VI. JUSTIFICATION OF TRANSACTION PATH DERIVATION

(Flows after the marginal increase in transacted power amount)

Line	Phase Layer A – pu	Phase Layer B – pu	Phase Layer C – pu	Neutral Layer – pu
AB	+3+2j (0)	+3+2j (0)	+3+2j (0)	0 (0)
BC	+2+1j (0)	+2+1j (0)	+2+1j (0)	0 (0)
AD	-0.1+2j (-0.1)	+0.1+2j (+0.1)	0+2j (0)	+0.15+j0.087 (+0.15+j0.087)
DE	+3+1j (0)	+3.1+1j (+0.1)	+3+1j (0)	+0.05+j0.087 (+0.05+j0.087)
DF	-4.1+1j (-0.1)	-4+1j (0)	-4+1j (0)	+0.1 (+0.1)

Bearing in mind the results of Table VI, the path derivation that was proposed in section III.*A* is effectively confirmed based on which line flows are altered when the transacted power amount is marginally increased. Furthermore, the direction of the transaction path is also confirmed by the sign of the flow changes. For example, considering line A_AD_A, the flow change is -0.1 pu. This is interpreted as an increase in the flow from node D_A to node A_A which is in agreement with the transaction's direction on phase layer A. On the other hand, considering line A_BD_B, the change in flow is +0.1 pu. This entails an increase in flow from node A_B to node D_B, which again is in agreement with the transaction direction on phase layer B. Therefore, this test validates the multi-layered approach that is proposed.

IV. PROPOSED LOSS ALLOCATION FRAMEWORK

The rationale of the proposed framework is to allocate losses to each transaction depending on its contribution to the real and reactive flows of each line that belongs to its *path*. The transactions' contribution in line flows and losses is dependent on a) the transacted real and/or reactive power amount (i.e. P_{TR} and Q_{TR} respectively) and b) their *direction* relative to each line's real and reactive flows (i.e. JP_e^k and JQ_e^k respectively). The fundamental logic of this loss allocation process is applied both for the phase layers as well as for the neutral layer. With regard to the latter, the main difference lies in that line power flows on the neutral layer are the result of the vector sum of the phase flows and, therefore, this fact should be incorporated in the allocation process.

The allocation process is performed sequentially for each line of the network. Therefore, it should be explicitly mentioned that the final amount of losses liable to a transaction is simply the sum of its allocated losses for all lines that it uses (i.e. both on the phase layer(s) as well as the neutral layer).

A. Allocating losses incurred on phase layers

Assume that a line on phase layer *k* connects nodes X_k and Y_k. In order to determine the amount of losses ($AL_{TR}^{X_kY_k}$) that should be allocated to the TR^{th} transaction for using this line, the formulation shown in (3) can be utilized.

$$AL_{TR}^{X_kY_k} = \frac{P_{TR}^{X_kY_k}}{JP_{X_kY_k}} \times Lp_{X_kY_k} + \frac{Q_{TR}^{X_kY_k}}{JQ_{X_kY_k}} \times Lq_{X_kY_k} \quad (3)$$

Within (3), the annotation X_kY_k is used to indicate the direction in which the TR^{th} transaction uses the line connecting nodes X_k and Y_k as this is derived from the transaction path (see section III). Specifically, the ratio $P_{TR}^{X_kY_k} / JP_{X_kY_k}$ is utilized in order a) to determine the *relative* contribution of the transacted real power amounts to the line's real power flow and b) derive whether that contribution is in the same or opposite direction to the line's flow. In particular, if the ratio $P_{TR}^{X_kY_k} / JP_{X_kY_k}$ is positive, then the transaction is debited for losses. Conversely, if the ratio $P_{TR}^{X_kY_k} / JP_{X_kY_k}$ is negative, then the transaction is credited for losses. The exact same logic applies for the reactive power flows (based on the ratio $Q_{TR}^{X_kY_k} / JQ_{X_kY_k}$).

To make the above process more explicit, the network in Fig. 2 and the particulars of Table IV are revisited. Moreover, the transactions of Table VII are considered. Within Table VII, it is assumed that the node F_A sells 2 units of real power to node A_A, 1 unit of real power to node E_A and 1 unit of real power to node E_B. Moreover, node A_A sells 1 unit of reactive power to node F_A. Each of these four transactions uses the line segment (F_AD_A).

According to Table IV, the real power flow through that line segment is 4 pu with direction from F_A to D_A whereas the reactive power flow through the same line is 1 pu in the opposite direction. Assuming further that $a_{F_AD_A} = 0.01\,\text{pu}^{-1}$, the incurred losses due to the real power flow through the line are 0.16 pu whereas the incurred losses due to the reactive power flow are 0.01 pu. Hence, the total incurred losses of that line segment are 0.17 pu. Via the application of (3), the allocated losses (in pu) to each transaction for using line (F_AD_A) are calculated in Table VII. As can be seen within Table VII, the total allocated losses to the transactions using line (F_AD_A) equal its total incurred losses.

TABLE VII. ALLOCATING LOSSES TO TRANSACTIONS USING LINE (F_AD_A)

Trans-action	P_{TR} (pu)	Q_{TR} (pu)	*From Node*	*To Node*	$JP_{F_AD_A}$ (pu)	$JQ_{F_AD_A}$ (pu)	$AL_{TR}^{F_AD_A}$ (pu)
1	2	0	F_A	D_A	4	-1	0.08
2	1	0	F_A	D_A			0.04
3	1	0	F_A	D_B			0.04
4	0	1	D_A	F_A			0.01

B. Allocating losses incurred on the neutral layer

The manner in which the neutral layer losses are allocated is, in principle, the same as that of the phase layer losses. However, the phase shift of each phase layer must be taken into account in order to appropriately allocate the losses incurred on the neutral conductors. This is achieved via the use of an auxiliary

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multiplier (i.e. the $LayerM^k$ multiplier shown in (4)) that appropriately orients the transaction's flows in order to compare them with the line flows of the neutral layer.

$$LayerM^k = \begin{Bmatrix} 1, & \text{for Phase Layer A} \\ e^{-j120}, & \text{for Phase Layer B} \\ e^{-j240}, & \text{for Phase Layer C} \end{Bmatrix} \tag{4}$$

Specifically, the formulation shown in (5) is used to determine the losses amount ($AL_{TR}^{X_N Y_N}$) that are allocated to the TR^{th} transaction for using the neutral layer [25].

$$\begin{aligned} AL_{TR}^{X_N Y_N} &= A + B \quad \text{where} \\ A &= \frac{\operatorname{Re}\{(P_{TR}^{X_k Y_k} + jQ_{TR}^{X_k Y_k})^* \times LayerM^k\}}{JP_{Y_N X_N}} \times Lp_e^N \\ B &= \frac{\operatorname{Im}\{(P_{TR}^{X_k Y_k} + jQ_{TR}^{X_k Y_k})^* \times LayerM^k\}}{JQ_{Y_N X_N}} \times Lq_e^N \end{aligned} \tag{5}$$

We now take Fig. 2 and Table VI as our starting point in order to study the flow of unbalanced power through the neutral conductor. To this end, the transactions shown in Table VIII are examined. Taking into account that line ($F_N D_N$) exhibits a 0.1 pu power flow (as derived in Table VI) and assuming further that $a_{F_N D_N} = 0.01$pu^{-1}, the losses incurred through that line segment are 0.0001 pu. Via the application of (5), we get the results shown in Table VIII. Within Table VIII, it is shown that the allocated losses to all transactions using line ($F_N D_N$) sum up to 0.0001 pu, i.e. (5) yields total incurred losses on the conductors of the neutral layer.

TABLE VIII. ALLOCATING LOSSES TO TRANSACTIONS USING LINE ($F_N D_N$)

Trans-Action	P_{TR} (pu)	Q_{TR} (pu)	*From Node*	*To Node*	$JP_{D_N F_N}$ (pu)	$JQ_{D_N F_N}$ (pu)	$AL_{TR}^{F_N D_N}$ (pu)
1	2	0	F_A	D_A	0.1	0	0.002
2	1	0	F_A	D_A			0.001
3	1.1	0	F_A	D_A			0.0011
4	0	1	D_A	F_A			0
5	4	-1	F_B	D_B			-0.0011
6	4	-1	F_C	D_C			-0.0029

V. CASE STUDY

The following case study pertains to a systematic review of the proposed loss allocation framework based on the characteristics of the distribution network in Cyprus [11]. The Cypriot power system is currently experiencing a rapid growth in DG applications, mainly due to the island's high solar potential and due to the implementation of the Net Energy Metering (NEM) scheme [37]. To this extent, it provides a suitable test bed for capturing the implications of the DG impact on incurred losses and their subsequent allocation.

A. Test network description and data assumptions

Fig. 3 demonstrates the single-line diagram of an actual 3Ph-4W 400/230-V feeder that is considered representative of the LV distribution system characteristics in Cyprus [11]. The feeder serves 36 single-phase residential and 2 three-phase commercial customers (located at nodes 3 and 5). All 36 residential customers are uniformly distributed among the three phases of the feeder.

The feeder's coincident peak demand is 48 kW and occurs at the afternoon hours during the summer period. In terms of DG penetration, the network is assumed to benefit from 18 kW$_p$ of installed PV capacity (6 PV systems rated at 3 kW$_p$ each, assumed to generate at their maximum point at the time of peak demand). These systems are located at the premises of 6 single-phase residential customers, i.e. prosumers. The details of the DG penetration and customers' net demand are provided within Fig. 4. With regard to reactive power requirements, it is assumed that loads operate with a lagging power factor (cosφ) equal to 0.95.

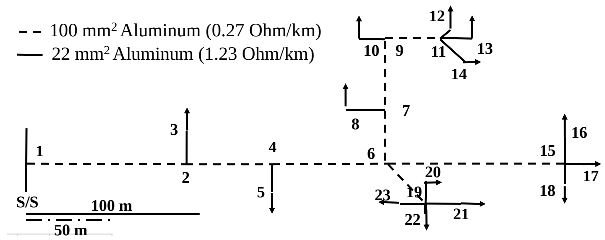
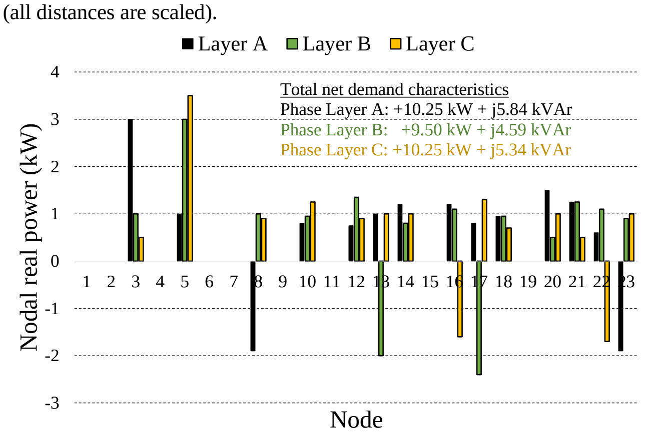


Fig. 3: Single-line diagram of an actual 3Ph-4W LV feeder located in Cyprus (all distances are scaled).



DG systems	Phase Layer A		Phase Layer B		Phase Layer C	
Location	Node 8_A	Node 23_A	Node 13_B	Node 17_B	Node 16_C	Node 22_C
Demand (kW+jkVAr)	+1.1+j0.36	+1.1+j0.36	+1+j0.33	+0.6+j0.2	+1.4+j0.46	+1.3+j0.43
Generation (kW+jkVAr)	-3+j0	-3+j0.5	-3+j0	-3-j0.5	-3+j0	-3+j0
Nodal power (kW+jkVAr)	-1.9+j0.36	-1.9+j0.86	-2+j0.33	-2.4-j0.30	-1.6+j0.46	-1.7+j0.43

Fig. 4: Customers' net demand characteristics at the time of peak demand.

B. Evaluation of incurred losses under balanced and unbalanced conditions

In order to demonstrate the importance of capturing the unbalanced nature of LV loads and DG, we examine the incurred losses as per two cases; in the first case it is assumed that the feeder is operating under ideal, balanced conditions. That is, all customers are connected in a three-phase manner and import/export the same power from each phase layer. Subsequently, in the second case, we assume that the feeder is operating under the actual, unbalanced conditions (as per Fig. 4). The incurred losses per each case and each layer are shown in Table IX.

TABLE IX. LOSSES FOR EACH CASE AND EACH LAYER

Case	Phase Layer A (W)	Phase Layer B (W)	Phase Layer C (W)	Neutral Layer (W)	Total Losses (W)
Balanced	116.5	116.5	116.5	0	349.5
Unbalanced	131.8	108	136.7	41.9	418.4
Difference (%)	+11.6%	-7.9%	+14.8%	+100%	+16.5%

According to Table IX, the actual unbalanced conditions entail 16.5% more losses than the case when the system is assumed to operate under balanced conditions. This result indicates that simplifying assumptions in terms of net demand unbalances at the LV level may introduce significant error in the calculated losses and, consequently, to their subsequent allocation. Furthermore, 10% of the total losses are incurred on the neutral layer. Therefore, omitting the influence of neutral flows on total losses (see subsection V.B.1) may introduce non-negligible errors in the loss allocation process.

1) Sensitivity of neutral losses to unbalanced conditions

In order to highlight the importance of taking into account the unbalanced conditions of 3Ph-4W networks, a concise sensitivity analysis is performed for the feeder under study (i.e. Fig. 3). In particular, three additional cases (to the main case study) are examined with regard to the incurred losses under different nodal DG characteristics. The assumptions (i.e. nodal DG power) as well as the incurred losses are shown in Table X. An interesting upshot of this sensitivity analysis is the fact that increasing DG penetration can potentially reduce total losses quite substantially (see Case 3). However, as DG penetration increases, neutral losses are also increasing. Therefore, the actual impact of DG units on the incurred losses may be positive in terms of reduced phase losses; however, this positive impact is counteracted by increased losses on the neutral layer due to the increased use of the neutral layer caused by DG units [35].

TABLE X: SENSITIVITY ANALYSIS RESULTS

	DG on Phase Layer A (kW+jkVAr)		**DG on Phase Layer B (kW+jkVAr)**		**DG on Phase Layer C (kW+jkVAr)**		**Total losses (W)**	**Neutral losses (% of total losses)**
Node	8_A	23_A	13_B	17_B	16_C	22_C	–	–
Case 1	-3	-6+j0.5	0	-3-j0.5	-3	-3	491.9	16.1%
Case 2	-3	-6+j0.5	0	-3-j0.5	0	-6	517.1	17.3%
Case 3	-6	-6+j0.5	-6	-6-j0.5	-6	-6	374.1	38.3%

C. Allocating losses to peer-to-peer transactions

As shown in the previous section, the balanced or unbalanced conditions for the feeder will inevitably have an impact on the loss calculation and allocation process. For example, if balanced conditions are assumed, then a transaction may be allocated more or less losses compared to the unbalanced conditions.

To this end, we examine this implication next by assuming that the peer-to-peer transactions of Table XI take place. With reference to Table XI, the first transaction pertains to a single-phase prosumer feeding a portion of his exported power to a single-phase consumer located on the same layer. This may be perceived as the most straightforward case of peer-to-peer transactions. Subsequently, the second transaction entails a single-phase prosumer feeding a single-phase consumer located on a different layer. The third transaction reflects the exchange of reactive power between two single-phase prosumers located at different layers. Finally, the fourth, fifth and sixth transactions pertain to the case where multiple single-phase prosumers located at different layers feed power to a three-phase consumer. The latter exercise is undertaken in order to demonstrate that the essential difference between three-phase and single-phase consumers (and/or prosumers) is simply the fact that three-phase customers are connected to multiple layers [32]. However, the fundamental approach of the proposed allocation process can appropriately handle such cases by treating the import/export activity of three-phase customers on each phase layer separately.

The allocated losses to each peer-to-peer transaction of Table XI are provided in Fig. 5. Within Fig. 5, it is shown that the balanced case entails substantially different results compared to the actual, unbalanced conditions of the feeder. Specifically, if balanced conditions are assumed, all transactions in Table XI (except the second) would be credited for losses. In other words, transactions 1 and 3-6 would be assumed to contribute to reducing the losses of the feeder. However, this is not the case under unbalanced conditions. Based on Fig. 5, the actual grid interaction of the peer-to-peer transactions tends to increase total losses and, therefore, they are debited –not credited– for their contribution. The only exception is the fourth transaction which is credited for losses even under unbalanced conditions; however, the respective credit is rather modest compared to the balanced case.

TABLE XI. PEER-TO-PEER TRANSACTIONS

TR	***P_{TR}* (kW)**	***Q_{TR}* (kVAr)**	***From Node***	***To Node***	***Description***
1	1	0	16_C	23_C	Single-phase prosumer to single-phase consumer on the same layer
2	1	0	13_B	13_A	Single-phase prosumer to single-phase consumer on different layers
3	0	0.3	17_B	23_A	Single-phase prosumer to single-phase prosumer on different layers
4	1	0	23_A	5_A	Multiple single-phase prosumers to three-phase consumer
5	1	0	13_B	5_B	
6	1	0	22_C	5_C	

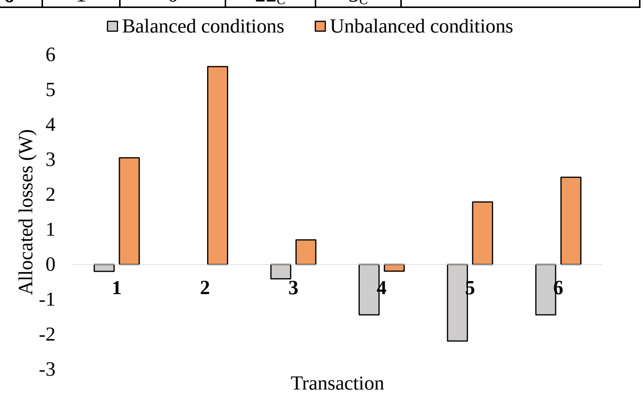


Fig. 5: Allocated losses to peer-to-peer transactions of Table XI for balanced and unbalanced conditions.

The differences between the two cases are dependent on the fact that the true topologies of 3Ph-4W distribution networks are principally deviant from balanced three-phase networks. This is particularly exemplified in the second peer-to-peer transaction. For this transaction, the actual path from the sending node to the receiving node includes lines (1-2), (2-4), (4-6), (6-7), (7-9), (9-11) and (11-13) for two phase layers (A and B) as well as the neutral layer. Therefore, the final amount of allocated losses would be determined based on the contribution of the transaction to the losses of each line of the three involved layers. However, if we assume that all customers are connected in a three-phase, balanced manner to the feeder, then the second transaction of Table XI would be allocated no losses (according to Fig. 5). The reason for this lies in the fact that if the buyer (i.e. consumer) and the seller (i.e. prosumer) were both connected to the same nodes of the network in an ideal, balanced manner, then they could directly exchange

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services without their transaction using the grid. This is an important implication that should be borne in mind in order to ensure that the allocation process is both consistent with actual power flows as well as cost-reflective.

D. *Allocating losses to Energy Communities*

Having shown the applicability and implications of the proposed loss allocation framework to peer-to-peer transactions, a further step is taken in order to discuss another emerging concept pertaining to *Energy Communities* (ECs).

Even though ECs settings closely resemble peer-to-peer transactions, their fundamental difference lies in the determination of the sending and receiving nodes. In particular, an EC entails that a specific group of nodes exchanges power with another group of nodes. Therefore, the arising question in terms of loss allocation is: *how should losses be allocated to groups of prosumers and consumers?*

In this regard, the proposed loss allocation framework can be appropriately adapted in order to integrate ECs. Specifically, a series of explicit step is followed, as discussed below:

1. Firstly, it is assumed that the point of connection between the upstream and local grid is the reference node for all transactions.
2. Then, the allocated losses to each nodal power absorption/injection of the feeder are calculated based on (3)-(5) and assuming that the reference node is a) the sending node for all demand imports and b) the receiving node for all DG exports.
3. Finally, the allocated losses of each node are used in order to determine the amount of losses that is liable to an EC by determining whether that node belongs to the EC or not.

To make the above process more explicit, we revisit the particulars of the feeder shown in Fig. 3 and assume that an EC is formed by a number of the feeder's customers. Specifically, the details of Table XII are assumed. According to Table XII, the EC consists of 6 single-phase prosumers (located at nodes 8_A, 23_A, 13_B, 17_B, 16_C and 22_C), 4 single-phase consumers (located at nodes 20_A, 20_B, 17_C and 18_C) and one three-phase consumer (connected at nodes 5_A, 5_B and 5_C). According to Table XII, the net real demand of the EC equals zero; this, however, does not entail that the use of the grid is avoided. Moreover, the EC is a net importer in terms of reactive power.

TABLE XII. ENERGY COMMUNITY DETAILS

	Phase Layer A	Phase Layer B	Phase Layer C
Group of nodes	5_A, 8_A, 20_A, 23_A	5_B, 13_B, 17_B, 20_B	5_C, 16_C, 17_C, 18_C, 22_C
EC demand per layer (kW+jkVAr)	+4.7+j1.5	+5.1+j1.7	+8.2+j2.7
EC generation per layer (kW+jkVAr)	-6+j0.5	-6-j0.5	-6+j0
EC net demand per layer (kW+jkVAr)	-1.3+j2	-0.9+j1.2	+2.2+j2.7

The objective of this exercise is to quantify amount of losses that the EC (as a whole) must procure from the loss supplier (i.e. the upstream market) in order to compensate for its contribution to the feeder's total incurred losses. To facilitate the latter, Table XIII provides the allocated losses to each node according to steps 1 and 2 above.

The nodes that are members of the EC are marked with a light grey color. By using these results of Table XIII, the allocated losses to the EC sum up to 64.6 W in total whilst the rest of the feeder's customers are allocated 353.9 W.

TABLE XIII: ALLOCATED LOSSES TO EACH NODE OF THE FEEDER IN FIG. 3

Node	Layer A		Layer B		Layer C	
	TAL_n^A (W)	$TAL_{n,A}^N$ (W)	TAL_n^B (W)	$TAL_{n,B}^N$ (W)	TAL_n^C (W)	$TAL_{n,C}^N$ (W)
1	0	0	0	0	0	0
2	0	0	0	0	0	0
3	22.0	4.9	5.8	-0.7	3.0	-0.5
4	0	0	0	0	0	0
5	8.6	-0.9	27.7	0.3	36.2	2.8
6	0	0	0	0	0	0
7	0	0	0	0	0	0
8	-13.3	2.4	11.1	0.8	11.5	0.6
9	0	0	0	0	0	0
10	9.8	0.0	10.7	-1.0	17.1	1.3
11	0	0	0	0	0	0
12	9.2	0.1	15.0	-1.6	12.1	0.9
13	12.6	0.5	-15.2	4.9	13.7	1.9
14	15.4	0.5	9.0	-1.2	13.8	1.1
15	0	0	0	0	0	0
16	15.6	3.9	11.5	-2.2	-11.9	4.3
17	10.2	2.1	-20.2	6.3	15.7	0.5
18	12.2	2.3	9.8	-2.0	8.4	-0.2
19	0	0	0	0	0	0
20	17.8	1.4	5.6	-0.1	12.0	-0.7
21	14.4	0.9	14.1	0.3	5.8	-0.5
22	6.9	0.8	12.6	0.5	-12.9	3.0
23	-9.7	3.4	10.4	1.5	12.3	-0.7
Total (W)	131.8	22.2	108.0	5.8	136.7	13.9

VI. CONCLUSIONS

The concepts of distributed TE market environments are currently drawing increased interest from involved stakeholders. The transition from the traditional, centralized power systems' regime towards a more distributed status quo is giving rise to several opportunities as well as challenges.

Within that context, this paper offers a solution to the loss allocation issues of TE environments by harmonizing a) the physical aspects of radial or tree distribution networks with b) the financial transactions (e.g. *peer-to-peer* trades) that may take place. The method is shown to yield total incurred losses by taking into account the contribution of each transaction to the incurred losses of those parts of the network that are used to complete it.

The regulatory implications that are instigated from this work pertain to the fact that calculating and allocating losses assuming that LV feeders are operating under balanced conditions may result in erroneous results and significant error that inevitably distorts the allocation process.

Moreover, the incorporation of bilateral, peer-to-peer transactions on the same phase layer but also between nodes located at different layers was achieved via the representation of 3Ph-4W as multi-layered graphs. This was particularly important for accurately deriving the allocated losses of each transaction.

Finally, the proposed framework was also shown to be capable of facilitating the transactions of groups of customers forming an *Energy Community*. To this extent, the flexible nature of the proposed loss allocation framework provides a suitable candidate-solution for future TE markets.

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